COO-3077-14
MF-76

Courant Institute of
Mathematical Sciences

Magneto-Fluid Dynamics Division

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AEC Research and Development Report

Plasma Physics
August 1972

New York University

COO-3077-14 c.3 (microfiche)

UNCLASSIFIED

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STABILITY OF LONG HELICAL WAVELENGTH FREE BOUNDARY EQUILIBRIA WITH SLIGHTLY ELLIPTICAL CROSS SECTION

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U. S. Atomic Energy Commission
Contract No. AT(11-1)-3077

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Abstract

Free boundary helically symmetric ideal magnetohydrodynamic equilibria are obtained which correspond to a system with L = 1 plus L = 2 magnetic fields. The configuration most stable to m = 1 modes has L = 2 source fields only in the plasma and is stable to these modes with conducting walls at any finite position.

I. Introduction

This report contains the details of computations completed about two years ago and previously reported.[1,2] In an earlier paper,[3] we examined free boundary, long helical wavelength ideal magnetohydrodynamic equilibrium where the plasma column had a circular cross section. A careful examination of the stability analysis given there indicated that the stability properties of the equilibrium were strongly affected by the small admixture of $L = 2$ helically symmetric fields in the basically $L = 1$ system. It is thus of some interest to generalize the previous study and admit a small but variable mixture $L = 2$ fields and discover their effect on the equilibrium and its stability. The effect of the addition of these fields is to change the plasma vacuum interface from a circle to an ellipse of small eccentricity. We generate such equilibria and we obtain the stability properties to modes which are principally $m = 1$. We find that the most stable equilibria do not have a circular cross section, but are slightly elliptical and that these equilibria are neutrally stable with a conducting wall at infinity and stable with a conducting wall at any finite position. An equivalent characterization of these equilibria is that they have no $L = 2$ source fields at infinity, but only $L = 2$ sources arising from the plasma currents. These results are obtained on the basis of expansions detailed below, but they indicate that any finite deformation theories should admit non-circular plasma cross sections to insure the physical significance of the conclusions.

The basic expansion parameter in this process is the ratio of plasma radius to helical wavelength. We follow the notation

and procedure of the earlier work[3] in detail. We obtain an equilibrium with finite deformation and then make an additional approximation of small deformation to compute δW.

II. Equilibrium

We pick up the specification of the equilibrium from I at equation 8 and we give a different flux function $\psi(\bar{x},\bar{y})$ inside the plasma interface. We set

$$\psi(\bar{x},\bar{y}) = \frac{\gamma}{2}\{\bar{x}^2 + \bar{y}^2 + 2a(1-c)\bar{x} + 2b(1+c)\bar{y} - c(\bar{x}^2 - \bar{y}^2)\} ,$$

where $0 \leq c < 1$ and a and b measure the admixture of L = 1 fields and c gives the mixing of L = 2 fields. If we set

$$x' = \bar{x} + a$$

$$y' = \bar{y} + b$$

then

$$\psi(x',y') = \frac{\gamma}{2}((1-c)x'^2 + (1+c)y'^2) + \text{constant.}$$

We work in elliptical coordinates defined by

$$x' = \sqrt{2c}\ \cosh\ \xi\ \cos\ \eta$$

$$y' = \sqrt{2c}\ \sinh\ \xi\ \sin\ \eta$$

and we define ξ_o by

$$\cosh\ \xi_o = \sqrt{\frac{1+c}{2c}}$$

so that $\psi(\xi,\eta)$ is constant on $\xi = \xi_o$. Thus $\xi = \xi_o$ is a flux surface, which we take to be the plasma vacuum interface. A direct computation shows that inside the interface

$$\underset{\sim}{B} = \gamma\{\varepsilon\,\hat{x}'(cy' + b) + \varepsilon\,\hat{y}'(cx' - a) + \hat{z}(1 + O(\varepsilon^2))\}$$

and that

$$B^2 = \gamma^2\{1 - \varepsilon^2(a^2 + b^2 + 2c^2)\} + \varepsilon^2\gamma^2\{2\sqrt{2c}\,(a\cosh\xi\cos\eta + b\sinh\xi\sin\eta)$$

$$+ c^3(\cosh 2\xi + \cos 2\eta) - 2c^2\cosh 2\xi\cos 2\eta\}\ ,$$

while on $\xi = \xi_o$

$$B^2 = \gamma^2\{1 - \varepsilon^2(a^2 + b^2 + c^2)\} + \varepsilon^2\gamma^2\{2a\sqrt{1+c}\cos\eta + 2b\sqrt{1-c}\sin\eta$$

$$+ c^3\cos 2\eta - 2c\cos 2\eta\}\ .$$

If we generate the magnetic field outside the plasma from a flux function ψ and constant B_z field of magnitude ζ, then B^2 outside is given by (c.f. I-6)

$$B^2 = \zeta^2 + \varepsilon^2\{\frac{(\psi_{,\xi})^2 + (\psi_{,\eta})^2)}{c(\cosh 2\xi - \cos 2\eta)} - \zeta^2(c(\cosh 2\xi + \cos 2\ )$$

$$-2a\sqrt{2c}\ \cosh\xi\cos\eta - 2b\sqrt{2c}\sinh\xi\sin\eta + a^2 + b^2)\}$$

where

$$\nabla^2\psi = 2\zeta, \text{ or } \psi_{,\xi\xi} + \psi_{,\eta\eta} = 2\zeta c(\cosh 2\xi - \cos 2\eta).$$

For the surface $\xi = \xi_o$ to be a flux surface we require $\psi(\xi_o, \eta)$

to be constant, while pressure balance demands that the jump in B^2 be constant, where

$$[B^2] = \zeta^2 - \gamma^2 - \varepsilon^2(\zeta^2(a^2+b^2+1) - \gamma^2(a^2+b^2+2c^2)$$

$$+ \varepsilon^2\left[\frac{(\psi,\xi)^2 + (\psi,\eta)^2}{c(\cosh 2\xi - \cos 2\eta)} - \zeta^2 c(\cos h2\xi + \cos 2n)(1 + c^2(1-\beta))\right.$$

$$+ 2a\zeta^2\beta\sqrt{2c}\cosh\xi\cos\eta + 2b\,\zeta^2\beta\sqrt{2c}\sinh\xi\sin\eta$$

$$\left. + 2c^2\gamma^2\cosh 2\xi\cos 2\eta\right]$$

and

$$\zeta^2 - \gamma^2 = \beta\,\zeta^2 .$$

Hence on $\xi = \xi_o$ we infer that there is a constant K such that

$$\psi,\xi\,|_{\xi=\xi_o} = \zeta\{(1 - c\cos 2\eta)(K - 2\alpha\beta\sqrt{1+c}\cos\eta - 2b\beta\sqrt{1-c}\sin\eta$$

$$-c(\cos 2\eta)\,(1 - 2\beta - c^2(1-\beta))\}^{1/2}$$

It is convenient to introduce $\tilde{a} = \beta\alpha\sqrt{1+c}$, $\tilde{b} = \beta b\sqrt{1-c}$ so that

$$\psi,\xi\,|_{\xi=\xi_o} = \zeta\{(1 - c\cos 2\eta)(K - 2\tilde{a}\cos\eta - 2\tilde{b}\sin\eta$$

$$-c(\cos 2\eta)\,(1 - 2\beta - c^2(1-\beta))\}^{1/2} .$$

We can now determine ψ as

$$\psi(\xi,\eta) = \zeta \frac{c}{2}(\cosh 2\xi + \cos 2\eta) + \zeta\left[\sum_{n=1}^{\infty}(\alpha_n \cos n\eta + \beta_n \sin n\eta)\right.$$

$$\left.\frac{\sinh n(\xi - \xi_o)}{n}\right] + \zeta\alpha_o\xi - \frac{\zeta c}{2}\cos 2\eta \cosh 2(\xi - \xi_o)\ ,$$

where the various constants are given by

$$\alpha_o + \sqrt{1-c^2} + \sum_{n=1}^{\infty}(\alpha_n \cos n\eta + \beta_n \sin n\eta) =$$

$$\sqrt{(1 - c\cos 2\eta)(K - 2a\cos\eta - 2b\sin\eta}$$

$$\overline{-(\cos 2\eta)(1 - 2\beta - c^2(1-\beta)))}$$

We shall ultimately expand assuming that $\tilde{a}$ and $\tilde{b}$ are small and of the same order and that c is of order $\tilde{a}^2$ (or $\tilde{b}^2$). We note that the net z pinch current is essentially $\sqrt{K} - 1$. Thus, we take $\sqrt{K} = 1 + \ell$, where $\ell = O(\tilde{a}^2) = O(c)$. At this point we keep K and only use the ordering assumption to drop unnecessary terms. If we expand $\psi_{,\xi}$ to order $c^2 =$ order $\tilde{a}^4$ and drop terms in $O(c^2)$ with angular dependence, we find

$$\psi_{,\xi} = \zeta\{\sqrt{K} - \frac{\tilde{a}^2+\tilde{b}^2}{4K\sqrt{K}} - \frac{c^2\beta^2}{4} - \frac{15}{64}(\tilde{a}^2+\tilde{b}^2)^2 + \frac{c}{8}(\tilde{a}^2-\tilde{b}^2)(3\beta - 1)$$

$$- \frac{(\tilde{a}+i\tilde{b})e^{-i\eta} + (\tilde{a}-i\tilde{b})e^{i\eta}}{2\sqrt{K}} - \frac{c(1-\beta)}{2}(e^{2i\eta} + e^{-2i\eta})$$

$$- \frac{1}{8}((\tilde{a}-i\tilde{b})^2 e^{2i\eta} + (\tilde{a}^2+i\tilde{b})^2 e^{-2i\eta}) + \frac{\beta c}{4}(\tilde{a}+i\tilde{b})(e^{i\eta} + e^{-3i\eta})$$

$$+ \frac{\beta c}{4}(\tilde{a}-i\tilde{b})(e^{-i\eta} + e^{3i\eta})$$

$$- \frac{1}{16}\left[(\tilde{a}+i\tilde{b})^3 e^{-3i\eta} + 3(\tilde{a}^2+\tilde{b}^2)\left((\tilde{a}+i\tilde{b})e^{-i\eta} + (\tilde{a}-i\tilde{b})e^{i\eta}\right) + (\tilde{a}-i\tilde{b})^2 e^{3i\eta}\right]\Big\} .$$

We shall also need

$$\frac{\partial}{\partial\xi}\left[\frac{B^2}{2}\right]\Big|_{\xi=\xi_o} = \varepsilon^2\Big\{\frac{\psi_{,\xi\xi}\psi_{,\xi}}{(1 - c\cos 2\eta)} - \frac{(\psi_{,\xi})^2 c \sinh 2\xi_o}{(1 - c\cos 2\eta)^2}$$

$$- \zeta^2 c \sinh 2\xi_o(1 + c^2(1-\beta) + \zeta^2 \tilde{a}\sqrt{\frac{2c}{1+c}}\sinh \xi_o \cos\eta$$

$$+ \zeta^2 \tilde{b}\sqrt{\frac{2c}{1-c}}\cosh \xi_o \sin\eta + 2c(1-\beta)\cos 2\eta\Big\}$$

$$= -\ell^2 - \frac{\tilde{a}^2+\tilde{b}^2}{2\sqrt{K}} - \frac{c^2\beta^2}{2} - \frac{15}{32}(\tilde{a}^2+\tilde{b}^2)^2 + \frac{c}{4}(\tilde{a}^2-\tilde{b}^2)(3\beta-1)$$

$$+ \tilde{a}\cos\eta + \tilde{b}\sin\eta - 2\beta c\cos 2\eta - \frac{1}{4}\left((\tilde{a}+i\tilde{b})^2 e^{-2i\eta} + (\tilde{a}-i\tilde{b})^2 e^{2i\eta}\right)$$

$$+ O(c^3) .$$

The terms omitted in the above expansion are either $O(c^5)$ or higher or their integral with respect to η is 0.

With these preliminaries we are ready to start the stability examination.

III Stability

We start the stability analysis by examining the Euler variational equations in the plasma region. As before

$$\underset{\sim}{\nabla}(\gamma p \underset{\sim}{\nabla}\cdot\underset{\sim}{\xi} - \underset{\sim}{B}\cdot\underset{\sim}{Q}) = -(\underset{\sim}{B}\cdot\underset{\sim}{\nabla})\underset{\sim}{Q} - (\underset{\sim}{Q}\cdot\underset{\sim}{\nabla})\underset{\sim}{B} ,$$

where

$$\underset{\sim}{Q} = \underset{\sim}{\nabla} \times (\underset{\sim}{\xi} \times \underset{\sim}{B}) .$$

At the risk of some ambiguity $\underset{\sim}{\xi}$ represents the usual plasma displacement and the usual elliptic coordinate. Context readily determines the meaning. We work in the x', y', z' coordinate system in which we readily find that

$$\underset{\sim}{B}\cdot\underset{\sim}{\nabla} = \varepsilon\gamma O = \varepsilon\gamma\{\frac{\partial}{\partial z'} + y'(1+c)\frac{\partial}{\partial x'} - x'(1-c)\frac{\partial}{\partial y'}\} .$$

We find that

$$\underset{\sim}{Q} = (\underset{\sim}{B}\cdot\underset{\sim}{\nabla})\underset{\sim}{\xi} - (\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi})\underset{\sim}{B} - (\underset{\sim}{\xi}\cdot\underset{\sim}{\nabla})\underset{\sim}{B} .$$

Since $\underset{\sim}{\nabla}\underset{\sim}{B} = O(\varepsilon)$ we see that we need $\underset{\sim}{Q}$ correct to order ε only so that we have

$$\underset{\sim}{Q} = \gamma\{\varepsilon O\underset{\sim}{\xi} - (\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi})\hat{z} - \varepsilon(\xi_{x'}\frac{\partial}{\partial x'} + \xi_{y'}\frac{\partial}{\partial y'})\left[\hat{x}'(cy'+b) + \hat{y}'(cx'-a)\right]\} + (O\varepsilon^2)$$

$$= \varepsilon\gamma\{O\underset{\sim}{\xi} - c(\xi_{x'}\hat{y}' + \xi_{y'}\hat{x}')\} - \gamma\hat{z}\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi} + O(\varepsilon^2)$$

$$= \varepsilon\gamma\{(O\xi_{x'} - (1+c)\xi_{y'})\hat{x}' + (O\xi_{y'} + (1-c)\xi_{x'})\hat{y}'\} - \gamma(\varepsilon O\xi z + (\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi}))\hat{z}' + O(\varepsilon^2)$$

Correct to order ε^2 we find

$$(\underset{\sim}{B}\cdot\underset{\sim}{\nabla})\underset{\sim}{Q}+(\underset{\sim}{Q}\cdot\underset{\sim}{\nabla})\underset{\sim}{B} = \varepsilon^2\gamma^2\{\hat{x}'(0^2\xi_{x'} - (1+c)0\xi_{y'} - 0\xi_{y'} - (1-c)\xi_{x'})$$

$$+ \hat{y}'(0^2\xi_{y'} + (1-c)0\xi_{x'} + 0\xi_{x'} - (1+c)\xi_{y'})$$

$$- \hat{z}'\varepsilon\gamma(\varepsilon 0^2\xi_z + 0(\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi}))$$

$$+ c\hat{y}'(0\xi_{x'} - (1+c)\xi_{y'}) + cx'(0\xi_{y'} + (1-c)\xi_{x'})\} ,$$

or

$$(\underset{\sim}{B}\cdot\underset{\sim}{\nabla})\underset{\sim}{Q}+(\underset{\sim}{Q}\cdot\underset{\sim}{\nabla})\underset{\sim}{B} = \varepsilon^2\gamma^2\{\hat{\underset{\sim}{x}}'(0^2\xi_{x'} - 20\xi_{y'} - (1-c)^2\xi_{x'})$$

$$+ \hat{\underset{\sim}{y}}'(0^2\xi_{y'} + 20\xi_{x'} - (1+c)^2\xi_{y'})\} \qquad (A)$$

$$- \varepsilon\gamma\hat{\underset{\sim}{z}}'(\varepsilon\, 0^2\xi_z + 0\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi}) .$$

Since the equilibrium is independent of z' we may take $\underset{\sim}{\xi}$ to have a dependence $e^{ikz'}$. We may then evaluate $\underset{\sim}{B}\cdot\underset{\sim}{Q} - \gamma p\underset{\sim}{\nabla}\cdot\underset{\sim}{\xi}$ correct to order ε^2 up to a constant multiple of $e^{ikz'}$, which will not enter any stability results, solely from the x' and y' components of (A). We may determine $\xi_{x'}$ and $\xi_{y'}$ from (A) by the equations

$$\frac{\partial}{\partial y'}(0^2\xi_{x'} - 20\xi_{y'} - (1-c)^2\xi_{x'}) = \frac{\partial}{\partial x'}(0^2\xi_{y'} + 20\xi_{x'} - (1+c)^2\xi_{y'}) \qquad (B)$$

and

$$\frac{\partial\xi_{x'}}{\partial x'} + \frac{\partial\xi_{y'}}{\partial y'} = 0$$

which gives

$$\gamma p \nabla \cdot \xi - B \cdot Q = \Omega$$

where

$$-\frac{\partial \Omega}{\partial x'} = (0^2 \xi_{x'} - 20\xi_{y'} - (1 - c^2 \xi_{x'})$$

$$-\frac{\partial \Omega}{\partial y'} = (0^2 \xi_{y'} + 20\xi_{x'} - (1 + c)^2 \xi_{y'}) \ .$$

We shall obtain the solutions corresponding to the m = $\pm$ 1, $\pm$ 2, $\pm$ 3 modes we need to obtain stability results.

We note that 0 is a differential operator which takes a homogeneous polynomial in x and y of degree n into a polynomial of the same type. Thus if we were to represent $\xi_{x'}$ and $\xi_{y'}$ by two homogeneous polynomials of degree n, involving 2n +2 arbitrary constants, by setting the coeficients of $x^p y^q$ to zero in (B), we would obtain 2n relations, indicating the existence of two independent solutions of this type. The solutions homogeneous of degree n correspond essentially to the modes m = $\pm$ |n +1|, as we shall shortly see. Rather than follow this cumbersome procedure, we shall employ the earlier stability work to infer the relative amplitudes of the modes and hence we shall scale the modes with a or c. To maintain the parallelism with (I) we take $\underset{\sim}{\xi}$ to be principally m = -1 with an admixture of m = -2 of order c, m = -3 of order c and m = $\pm$ 1 of order c. With this ordering we shall greatly simplify the work. We recall that also $\sqrt{K} = 1 + \ell$, where $\ell = 0(c)$ also. We proceed mode by mode.

A. $n = 0$ or $m = \pm 1$

We set

$$\xi_x = D\, e^{-ikz'}$$

$$\xi_y = E\, e^{-ikz'}$$

so that

$$\Omega_{,x} = \{(-ik)^2 D + 2ikE - (1 - c|^2 D\} e^{-ikz'}$$

$$\Omega_{,y} = \{(-ik)^2 E - 2ikD - (1 + c)^2 E\} e^{-ikz'}$$

or

$$\Omega = \{((k^2 + (1 - c)^2)D - 2ikE)x' + ((k^2 + (1 + c)^2)E + 2ikD)y'\} e^{-ikz}$$

while

$$\underset{\sim}{n} \cdot \underset{\sim}{\xi} = \frac{(D \sinh \xi_o \cos \eta + E \cosh \xi_o \sin \eta)}{\sqrt{\sinh^2 \xi_o + \sin^2 \eta}}$$

We see that $n = 0$ does indeed correspond to the $m = \pm 1$ modes, for which $\underset{\sim}{n} \cdot \underset{\sim}{\xi} = (e^{\pm i\eta} + \ldots)$. We may readily isolate the principal component by appropriate choice of D and E. For $c = 0$ Ω vanishes for

$$(k^2 + 1)D - 2ikE = 0$$

$$2ikD + (k^2 + 1)E = 0 ,$$

or

$$k = \frac{D}{iE} = \pm 1 \ .$$

We may then select the m = -1 mode by taking

$$D = 1 + \delta/2$$

$$E = -i(1 - \delta/2)$$

$$\mathcal{k}' = \mathcal{k} - 1$$

where $\mathcal{k}'$ and δ are of order c, so that

$$\Omega = O(c)e^{i\eta} + O(c)e^{-i\eta} + O(c^2)$$

and

$$\hat{n} \cdot \underset{\sim}{\xi} = e^{-i\eta} + O(c) \ .$$

We shall need the higher terms shortly.

B. n = 1 or m = $\pm$ 2

We shall assume that $\underset{\sim}{\xi}$ corresponding to n = 1 is O(c) and we obtain the leading terms by carrying over the results from I in the new notation. If we set

$$\xi_x = (Bx - Ay)e^{-ikz'}$$

$$\xi_y = (-Ax - By)e^{-ikz'}$$

(so that $\nabla\cdot\xi = \nabla \times \xi = 0$, as in I), and approximate Ω by its leading order in c

$$\Omega = \Omega_o + c\Omega_1 ,$$

$$\Omega_o = (-ik + y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}) ,$$

$$\Omega_1 = y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$$

we find

$$\Omega_{,x} = (k^2 + 4)\xi_x - 4ik\xi_y ,$$

$$\Omega_{,y} = 4ik\xi_x + (k^2 + 4)\xi_y ,$$

so that

$$\Omega = (k^2 + 4)(B\frac{x^2 - y^2}{2} - Axy) + 4ik(+\frac{Ax^2 - y^2}{2} + Bxy)$$

and hence

$$\Omega = \cos 2\eta(\frac{5}{2}B + 2iA) - \sin 2\eta(\frac{5A}{2} - 2iB) + O(c^2)$$

$$\underset{\sim}{n}\cdot\underset{\sim}{\xi} = B\cos 2\eta - A\sin 2\eta + O(c^2) .$$

Since reflection in the x,y plane leaves 0 unchanged we see that we may take $\underset{\sim}{\xi}$ as exactly even or odd and hence the error terms contain no $m = \pm 1, \pm 3$, etc. terms. Clearly, if we set $A = iB$, $\underset{\sim}{\xi}$ will represent the $m = -2$ mode.

C. $n = 2$ or $m = \pm 3$

We again take $\underset{\sim}{\xi}$ of order c, but now we must compute $\underset{\sim}{\xi}$ correct to order c^2, since there may be an admixture of $m = \pm 1$ in the higher order terms. To leading order in c, we again take over the old result in our new notation

$$\xi_x = (\psi(x^2 - y^2) + 2\chi xy)e^{-ikz'} + c^2\xi_x^1 e^{-ikz'}$$

$$\xi_y = (-2\psi xy + \chi(x^2 - y^2))e^{-ikz'} + c^2\xi_y^1 e^{-ikz'}$$

so that

$$\Omega = \Omega_o + c^2\Omega_1 = ((k^2+4)\psi - 6ik\chi)(\frac{x^3}{3} - xy^2) + (6ik\psi + (k^2+4)\psi)$$

$$(x^2y - y^3/3) + c^2\Omega_1$$

we find

$$-\Omega_{,x} = -\Omega_{o,x} + c(0_1 0_o + 0_o 0_1)\xi_x^o - 2c0_1\xi_y^o + 2c\xi_x^o - cD_x^o\xi_x^1$$

$$-\Omega_{,y} = -\Omega_{o,y} + c(0_1 0_o + 0_o 0_1)\xi_y^o + 2c0_1\xi_x^o - 2c\xi_y^o - cD_y^o\xi_y^1 \quad , \qquad (C)$$

where D_x^o, D_y^o are the leading order differential operators. We readily see

$$O_1(x^2 - y^2) = 0$$

$$O_1\, xy = x^2 + y^2 \ .$$

We shall see that the right hand side of (C) is a gradient with $\underset{\sim}{\xi}^1 \equiv 0$ if $\underset{\sim}{\xi}^o$ corresponds to a m = -3 mode with k = 1 + O(c) and thus introduction of $\underset{\sim}{\xi}^1$ corresponds to a redefinition of the constants ψ, χ. A direct evaluation shows

$$-(\Omega_{,x} - \Omega_{o,x}) = c\{(8\psi - 4ik\chi)(x^2 + y^2) + 2\psi(x^2 - y^2) + 4\chi xy\}e^{-ikz'} + cD_x^o\xi'_x$$

$$-(\Omega_{,y} - \Omega_{o,y}) = c\{(8\chi + 4ik\psi)(x^2 + y^2) - 2\chi(x^2 - y^2) + 4\psi xy\}e^{-ikz'} + cD_x^o\xi'_y$$

so that with $\psi = i\chi + O(c^2)$ (corresponding to m = -3) and k = 1 + O(c) we find that

$$e^{ikz'}\Omega_o = \frac{4}{3}\psi\ e^{-3i\eta} + 2\psi c\ e^{-i\eta} + O(c^2)\ e^{3in} + O(c^2)\ e^{-3in} + O(c^3)$$

$$e^{ikz'}\Omega_1 = -2c\psi\ e^{-i\eta} + O(c^3)$$

while

$$\hat{n}\cdot\underset{\sim}{\xi} = \left(\sqrt{2c}\ \sqrt{\sinh 2\xi_o + \sin\eta}\right)^{-1}\{\psi\ e^{-3i\eta} + O(c^2)\} \ .$$

Since Ω is no larger than O(c) we do not need the $O(c^2)$ terms in $\hat{n}\cdot\underset{\sim}{\xi}$.

We are now prepared to start the evaluation of δW. In our new coordinate system

$$ds = \frac{r_o}{\ell}\sqrt{2c}\sqrt{\sinh^2\xi_o + \sin^2\eta}\, d\eta\, dz'(1 + O(\varepsilon^2)).$$

We may then readily evaluate the inner volume term as

$$2\delta W^{vi} = \frac{r_o}{\ell}\sqrt{2c}\int d\eta \sqrt{\sinh^2\xi_o r \sin^2\eta}\, dz\, \overline{(\underset{\sim}{n}\cdot\underset{\sim}{\xi})}\quad \Omega$$

$$= 2\pi\varepsilon^2 L\zeta^2(1-\beta)\{\frac{c\sinh\xi_o}{2}[D\bar{D}(k^2+(1-c)^2) + 2ik(D\bar{E} - \bar{D}E)$$

$$+ (k^2 + (1+c)^2)E\bar{E}] + \frac{B\bar{B}}{2} + \frac{4}{3}\psi\bar{\psi}\}$$

$$= 2\pi\varepsilon^2 L\xi^2(1-\beta)\{k^2 + (\delta - c)^2 + \frac{B\bar{B}}{2} + \frac{4}{3}\psi\bar{\psi}\}\ .$$

To obtain the surface term we must evaluate $(n\cdot\xi)^2$ to order c. We have

$$\hat{n}\cdot\underset{\sim}{\xi} = \frac{(1+\frac{\delta}{2})\sinh\xi_o\cos\eta - i(1-\delta/2)\cosh\xi_o\sin\eta}{\sqrt{\sinh^2\xi_o + \sin^2\eta}}$$

$$+ Be^{-2i\eta} + \psi e^{-3i\eta}$$

$$= e^{-i\eta} + (\frac{\delta}{2} - \frac{c}{4})e^{i\eta} + Be^{-2i\eta} + (\psi + \frac{c}{4})e^{-3i\eta}$$

$$= e^{-i\eta}\{1 + (\frac{\delta}{2} - \frac{c}{4})e^{2i\eta} + Be^{-i\eta} + (\psi + \frac{c}{4})e^{-2i\eta}\}$$

therefore

$$|\hat{n}\cdot\underset{\sim}{\xi}|^2 = 1 + Be^{-i\eta} + \bar{B}e^{i\eta} + e^{2i\eta}(\frac{\delta}{2} + \bar{\psi}) + e^{-2i\eta}(\frac{\bar{\delta}}{2} + \psi).$$

We thus readily find

$$2\delta W^{\Sigma} = 2\pi\varepsilon^2 L\zeta^2\{-\ell^2 - \frac{\tilde{a}^2+\tilde{b}^2}{2K\sqrt{K}} - \frac{c^2\beta^2}{2} - \frac{15}{32}(\tilde{a}^2+\tilde{b}^2)^2 + \frac{c}{4}(\tilde{a}^2-\tilde{b}^2)(3\beta-1)$$

$$+ \frac{B(\tilde{a}-i\tilde{b})}{2} + \frac{\bar{B}(\tilde{a}+i\tilde{b})}{2} - (\frac{(\tilde{a}+i\tilde{b})^2}{4} + \beta c)(\frac{\delta}{2}+\bar{\psi}) \qquad (D)$$

$$- (\frac{(\tilde{a}-i\tilde{b})^2}{4} + \beta c)(\frac{\bar{\delta}}{2}+\psi)\} \ .$$

We now proceed to the intricate outer volume term. In order to solve for the perturbed scalar potential of the magnetic field outside the plasma we must evaluate

$$\frac{\partial\Phi}{\partial n} = \hat{\xi}\cdot\nabla\times(\xi\times B) \ .$$

To leading order in ε

$$\hat{\xi}\cdot\nabla\times(\xi\times B) = \hat{\xi}\cdot\{(B\cdot\nabla)\xi - (\nabla\cdot\xi)B - (\xi\cdot\nabla)B\}$$

$$= \hat{\xi}\cdot\{(B\cdot\nabla)\xi - (\xi\cdot\nabla)B\} \ ,$$

where

$$B = \zeta\hat{z} - \varepsilon\,\hat{z}\times\nabla(\psi - \frac{\zeta\rho^2}{2})$$

$$= \zeta(\hat{z} - \varepsilon a\hat{y} + \varepsilon\hat{z}b) - \varepsilon\zeta\hat{z}\times\nabla\Psi \ ,$$

and

$$\Psi = \psi/\zeta - (r')^2/2$$

If we set

$$\underset{\sim}{B}_1 = \varepsilon\zeta\hat{z} \times \underset{\sim}{\nabla}\Psi ,$$

then

$$\frac{\partial\Phi}{\partial n} = \Phi_1 + \Phi_2 + \Phi_3 ,$$

where

$$\Phi_1 = \varepsilon\zeta\hat{\xi}\cdot\{\frac{\partial}{\partial z'} + y'\frac{\partial}{\partial y'} - x'\frac{\partial}{\partial y'}\}\underset{\sim}{\xi}$$

$$\Phi_2 = \varepsilon\zeta\hat{\xi}\cdot(\underset{\sim}{B}_1\cdot\underset{\sim}{\nabla})\underset{\sim}{\xi}$$

$$\Phi_3 = -\varepsilon\zeta\hat{\xi}\cdot(\underset{\sim}{\xi}\cdot\underset{\sim}{\nabla})\underset{\sim}{B}_1$$

In the above formulas $\hat{\xi}$ refers to a unit vector of the elliptic coordinate system and $\underset{\sim}{\xi}$ is the usual displacement.

We may evaluate Φ_1 directly by inserting the expressions for $\underset{\sim}{\xi}$ above (and recalling $\frac{\partial}{\partial z'}\hat{x}' = \hat{y}'$, $\frac{\partial}{\partial z'}\hat{y}' = \hat{x}'$), to find

$$\Phi_1 = -i\varepsilon\zeta[\{\not{\lambda}'e^{-i\eta} + \delta e^{i\eta} - Be^{-2i\eta} - 2\psi e^{-3i\eta}\} + O(c^2)] \qquad (E)$$

Before obtaining Φ_1 and Φ_2 we determine the terms we need. Since we want δW correct to order c^2 we see that we need all terms in Φ_1 of order $\tilde{a}$ = order $\tilde{b}$, order $\tilde{a}^2$ = order $\tilde{b}^2$ = order c, and order $\tilde{a}c$. However, the only terms of order $\tilde{a}c$ that we need are those with the same angular dependence as the order $\tilde{a}$ terms.

We start by evaluating Ψ and its derivatives on the interface:

$$\Psi = \psi - r^2/2 = \psi - \frac{c}{2}\cos h2\xi - \frac{c}{2}\cos 2\eta$$

$$\Psi_{,\xi} = \psi_{,\xi} - c \sinh 2\xi_o = \psi_{,\xi} - 1 + O(c^2) \qquad \Psi_{,\xi\eta} = (\psi_{,\xi})_{,\eta}$$

$$\Psi_{,\xi\xi} = \psi_{,\xi\xi} - 2c \cosh 2\xi_o = -2c \cos 2\eta$$

$$\Psi_{,\eta} = \psi_{,\eta} + c \sin 2\eta = c \sin 2\eta$$

$$\Psi_{,\eta\eta} = \psi_{,\eta\eta} + 2 \cos 2\eta = 2c \cos 2\eta ,$$

and

$$\underset{\sim}{B}_1 = \frac{\hat{\xi}\Psi_{,\eta} - \hat{\eta}\Psi_{,\xi}}{\sqrt{c \cosh 2\xi - c \cos 2\eta}};$$

Finally we need some additional terms associated with properties of the elliptical coordinates (all evaluated at $\xi = \xi_o$ with error $O(c^2)$)

$$\sqrt{\cosh 2\xi - \cos 2\eta}\,\frac{\partial}{\partial\xi}\frac{1}{\sqrt{\cosh 2\xi - \cos 2\eta}} = -1 - c \cos 2\eta + O(c^2)$$

$$\sqrt{\cosh 2\xi - \cos 2\eta}\,\frac{\partial}{\partial\eta}\frac{1}{\sqrt{\cosh 2\xi - \cos 2\eta}} = -2c \sin 2\eta + O(c^2)$$

$$\hat{\xi}_{,\xi} = -c \sin 2\eta\, \hat{\eta} \qquad \hat{\eta}_{,\xi} = +c \sin 2\eta\, \hat{\xi}$$

$$\hat{\xi}_{,\eta} = \hat{\eta}(1 + c \cos 2\eta) \qquad \hat{\eta}_{,\eta} = -\hat{\xi}(1 + c \cos 2\eta) .$$

A direct evaluation yields

$$\Phi_3 = -\varepsilon\zeta\left(\frac{\xi_\xi \Psi_{,\xi\eta} + \xi_\eta \Psi_{,\eta\eta}}{(c\cosh 2\xi_o - c\cos 2\eta)} + \Psi_{,\eta}\left(\xi_\xi \frac{\partial}{\partial\xi} + \xi_\eta \frac{\partial}{\partial\eta}\right)\frac{1}{2(c\cosh 2\xi - c\cos 2\eta)}\right.$$

$$\left. + \frac{-\Psi_{,\xi}}{(c\cosh 2\xi_o - c\cos 2\eta)}[\xi_\xi\, c\sin 2\eta - \xi_\eta(1 + c\cos 2\eta)]\right\}$$

and ξ_ξ and ξ_η are the ξ and η components of the displacement ξ. Thus keeping terms only through $O(\varepsilon^3)$ we find

$$\sqrt{c\cosh 2\xi_o - c\cos 2\eta}\,\Phi_3 = -\varepsilon\zeta\{\xi_\xi \cdot \Psi_{,\xi\eta}(1 + \frac{c}{2}\cos 2\eta) + 2c(\cos 2\eta)\xi_\eta$$

$$-\xi_\xi\, c\sin 2\eta - \Psi_{,\xi}[\xi_\xi\, c\sin 2\eta - \xi_\eta(1 + \frac{3c}{2}\cos 2\eta)\} + O(c^2)$$

We must evaluate this combination for the $\underset{\sim}{\xi}$'s corresponding to $m = \pm 1$, $m = \pm 2$, $m = \pm 3$. We start with $m = \pm 1$. We recall

$$\underset{\sim}{\xi} = (1 + \delta/2)\hat{x} - i(1 - \delta/2)\hat{y}\ .$$

Thus

$$\xi_\xi = \frac{(1 + \delta/2)\sqrt{2c}\,\sinh \xi_o \cos\eta - i\sqrt{2c}\cosh\xi_o(\sin\eta)(1 - \delta/2)}{\sqrt{1 - c\cos 2\eta}}$$

$$\xi_\eta = \frac{-(1 + \delta/2)\sqrt{2c}\cosh\xi_o \sin\eta - i(1 - \delta/2)\sqrt{2c}\,\sinh\xi_o\cos\eta}{\sqrt{1 - c\cos 2\eta}}\ ,$$

and a direct evaluation gives

$$(1 + \frac{c}{2}\cos 2n)\xi_\xi = e^{-i\eta} + (\frac{\delta}{2})e^{i\eta} + \frac{c}{2}e^{-3i\eta} + O(c^2)$$

$$(1+\tfrac{3}{2}\cos 2\eta)\xi_\eta = (-i)\{e^{-i\eta} - (\tfrac{\delta - c}{2})e^{i\eta} + c\, e^{-3i\eta}\} + O(c^2) \ .$$

Hence

$$\sqrt{c\cosh 2\xi - c\cos 2\eta}\,\Phi_3^1 = -\varepsilon\zeta\{e^{-i\eta}(\Psi_{,\xi\eta} - i\Psi_{,\xi}) - 2i\, c\cos 2\eta e^{-i\eta}$$

$$- c\sin 2\eta\, e^{-i\eta} + \Psi_{,\xi\eta}(\tfrac{\delta}{2}\, e^{i\eta} + \tfrac{c}{2}\, e^{-3i\eta}) - i\Psi_{,\xi}(\tfrac{c-\delta}{2}e^{i\eta} + ce^{-3i\eta})$$

$$- \Psi_{,\xi}e^{-i\eta}\, c\sin 2\eta\}$$

We note that the only terms of order $\tilde{a}$ come from Φ_3^1 and from (E). We see that we need keep only those terms of order $\tilde{a}^3$ in

$\sqrt{c\cosh 2\xi - c\cos 2\eta}\,\Phi_3$ with angular dependence $e^{-2i\eta}$, and we find

$$\sqrt{c(\cosh 2\xi_o - \cos 2\eta)}\,\Phi_3^1 = -i\varepsilon\zeta[\{-\tfrac{e^{i\eta}}{2}(\tfrac{(\tilde{a}-i\tilde{b})^2}{4} + c(2-\beta))$$

$$-(\ell - \tfrac{\tilde{a}^2+\tilde{b}^2}{4K\sqrt{K}})e^{-i\eta} + e^{-2i\eta}(\tfrac{(\tilde{a}+i\tilde{b})}{\sqrt{K}} + \tfrac{c(1-\beta)(\tilde{a}-i\tilde{b})}{2}$$

$$+ \tfrac{3}{8}(\tilde{a}^2+\tilde{b}^2)(\tilde{a}+i\tilde{b})) + e^{-3i\eta}\,\tfrac{3}{2}(\tfrac{(\tilde{a}+i\tilde{b})^2}{4} - c\beta\} + O(c^2)$$

$$+ O(a^3) \text{ excluding } e^{-2i\eta} \ .$$

For the m = 2,3 terms we have to leading order

$$\sqrt{c\cosh\xi_o - c\cos 2\eta}\,\Phi_3^{2,3} = \varepsilon\zeta[\{\xi_\xi\Psi_{,\xi\eta} + \xi_\eta\Psi_{,\xi}\} + O(c^2)] \ .$$

For m = 2, ξ_ξ and ξ_η have angular terms $e^{\pm 2i\eta}$, thus to order $\tilde{a}c$

Φ_3^2 has no terms of the form $e^{-2i\eta}$ and we may set $\Phi_3^2 = 0$. Finally for Φ_3^3, we note $\xi_\xi = \psi e^{-3i\eta}$, $\xi_\eta = -i\psi e^{-3i\eta}$ so again no terms of the requisite angular dependence appear in order $\tilde{a}c$. Finally we may take $\Phi_3 = \Phi_3^1$ as given above.

The last term we need is Φ_2 and again we note that since $\underset{\sim}{B}_1$ has no z component $(B_1 \cdot \nabla)$ applied to ξ corresponding to $m = \pm 1$ has x and y components constant, is zero. Thus Φ_2 is of order $\tilde{a}c$ and we need evaluate $(B_1 \cdot \nabla)\underset{\sim}{\xi}$ only to leading order in the terms. Thus

$$\Phi_2 = -\hat{\xi} \cdot \{\Psi_{,\xi} \frac{\partial}{\partial \eta} \underset{\sim}{\xi}\} = + \Psi_{,\xi}(-\xi_{\xi,\eta} + \xi_\eta)$$

where we drop the $m = \pm 1$ terms in $\underset{\sim}{\xi}$. Again no $m = \pm 2$ contributes and we may take, correct to our order

$$\Phi_2 = -\psi(\tilde{a} - i\hat{b})i\, e^{-2i\eta} \ .$$

We thus find

$$\sqrt{1 - c \cos 2\eta}\,\frac{\partial \Phi}{\partial n} = \frac{\partial \Phi}{\partial \xi} = -\varepsilon\zeta i[+ e^{i\eta}\{- \frac{(\tilde{a} - i\tilde{b})^2}{8} - \frac{c}{2}(2 - \beta) + \delta\}$$

$$+ e^{-i\eta}\{k' - (\ell - \frac{\tilde{a}^2 + \tilde{b}^2}{4K\sqrt{K}})\} + e^{-2i\eta}\{\frac{\tilde{a} + i\tilde{b}}{\sqrt{K}} + \frac{c(1 - \beta)}{2}(\hat{a} - i\hat{b})$$

$$+ \frac{3}{8}(\hat{a}^2 + \hat{b}^2)(\hat{a} + i\hat{b}) + \psi(\hat{a} - i\hat{b}) - B\} + e^{-3i\eta}\{\frac{3}{2}(\frac{(\hat{a} + i\hat{b})^2}{4} - c\beta) - 2\psi\}].$$

Since Φ is harmonic

$$\Phi_{,\xi\xi} + \Phi_{,\eta\eta} = 0 + 0(\varepsilon^2) \ .$$

We include an outer conducter at approximate radius $\rho >> 1$ assuming $\frac{1}{\rho^2} \sim \tilde{a}$ and include the effects to leading order in the $m = \pm 1, -2$ terms and we find

$$2\delta W^{Ve} = \varepsilon^2\zeta^2 2\pi L\{|\delta - \frac{(\tilde{a} - i\tilde{b})^2}{B} - \frac{c}{2}(2 - \beta)|^2(1 + \frac{2}{\rho^2}) + |\mathcal{K}' - \mathcal{L} + \frac{\tilde{a}^2 + \tilde{b}^2}{4K\sqrt{K}}|(1 + \frac{2}{\rho^2})$$

$$+ \frac{1}{2}(1 + \frac{2}{\rho^4})|\frac{\tilde{a} + i\tilde{b}}{\sqrt{K}} + \frac{c(1 - \beta)}{2}(\tilde{a} - i\tilde{b}) + \frac{3}{8}(\tilde{a}^2 + \tilde{b}^2)(\tilde{a} + i\tilde{b}) \quad (F)$$

$$+ \psi(\tilde{a} - i\tilde{b}) - B|^2 + \frac{1}{3}|\frac{3}{8}(\tilde{a} + i\tilde{b})^2 - \frac{3c\beta}{2} - 2\psi|^2$$

We shall only add the effects of an outer conductor in the leading order terms. Before we write δW in its entirety, we note from (D) and (F) that no terms linear in ψ or B appear. Thus, we may take

$$\psi = B = 0$$

and just as before the mode studied is an admixture of $m = \pm 1$. With this simplification

$$2\delta W = 2\pi\varepsilon^2 L\zeta^2\{(1 - \beta)\{|\mathcal{K}'|^2 + |\delta - c|^2\} - \mathcal{L}^2 - \frac{\tilde{a}^2 + \tilde{b}^2}{2K\sqrt{K}} - \frac{c^2\beta^2}{2} - \frac{15}{32}(\tilde{a}^2 + \tilde{b}^2)^2$$

$$+ \frac{c}{4}(\tilde{a}^2 - \tilde{b}^2)(3\beta - 1) - (\frac{(\tilde{a} + i\tilde{b})^2}{8} + \frac{\beta c}{2})\delta - (\frac{(\tilde{a} - i\tilde{b})^2}{8} + \frac{\beta c}{2})\bar{\delta}$$

$$+ (1 + \frac{2}{\rho^2}) + |\delta - \frac{(\tilde{a} - i\tilde{b})^2}{8} - \frac{c}{2}(2 - \beta)|^2 + |\mathcal{K}' - \mathcal{L} + \frac{(\tilde{a}^2 + \tilde{b}^2)}{4K\sqrt{K}}|^2(1 + \frac{2}{\rho^2})$$

$$+ \frac{1}{2}(1 + \frac{2}{\rho^4})|\frac{(\tilde{a} + i\tilde{b})}{\sqrt{K}} + \frac{c(1 - \beta)}{2}(\tilde{a} - i\tilde{b}) + \frac{3}{8}(\tilde{a}^2 + \tilde{b}^2)(\tilde{a} + i\tilde{b})|^2$$

$$+ 3|\frac{(\tilde{a} + i\tilde{b})^2}{8} - \frac{c\beta}{2}|^2\} .$$

We recall that ℓ is an equilibrium parameter related to the net current in the z direction, J,

$$J = \ell - \frac{(\tilde{a}^2 + \tilde{b}^2)}{4} + O(\tilde{a}^4) \ ,$$

so that we may rewrite $2\delta W$ as

$$2\delta W = 2\pi\varepsilon^2 L\zeta^2 \{(1-\beta)k^2 + (k-J)^2\left(1+\frac{2}{\rho^2}\right) - J^2 + \frac{(\tilde{a}^2+\tilde{b}^2)^2}{32} + \frac{(\tilde{a}^2+\tilde{b}^2)}{\rho^4}$$

$$+ 2c^2(1-\beta/2)^2 + \frac{c}{2}(\tilde{a}^2-\tilde{b}^2)(1-\beta/2) + (2-\beta)\delta\bar{\delta}$$

$$- 2\mathrm{Re}[\delta(\frac{(\tilde{a}+i\tilde{b})^2}{4} + c(2-\beta)] + \frac{2}{\rho^2}|\delta - \frac{(\tilde{a}-i\tilde{b})^2}{8} - \frac{c}{2}(2-\beta)|^2\} \ .$$

Since

$$(2-\beta)\delta\bar{\delta} - 2\mathrm{Re}\ \delta(\frac{(\tilde{a}+i\tilde{b})^2}{4} + c(2-\beta)) + \frac{2}{\rho^2}|\delta - \frac{(\tilde{a}-i\tilde{b})^2}{8} - c(1-\beta/2)|^2$$

$$= (2-\beta)|\delta - \frac{\frac{(\tilde{a}-i\tilde{b})^2}{4} + c(2-\beta)}{2-\beta}|^2 - \frac{|\frac{(\tilde{a}-i\tilde{b})^2}{4} + c(2-\beta|^2}{2-\beta}$$

$$+ \frac{2}{\rho^2}|\delta - \frac{(\tilde{a}+i\tilde{b})^2}{8} - c(1-\beta/2)|^2$$

we infer that the minimum occurs for

$$\delta = \frac{(\tilde{a}-i\tilde{b})^2}{4(2-\beta)} + c$$

and the minimum of this term is

$$- \frac{|\frac{(\tilde{a}-i\tilde{b})^2}{4} + c(2-\beta)|^2}{2-\beta} + \frac{\beta^2}{2\rho^2}|\frac{(\tilde{a}-i\tilde{b})^2}{2-\beta} + c|^2$$

Thus

$$2\delta W = 2\pi\varepsilon^2\zeta^2 L\{(1-\beta)\mathcal{K}' + (\mathcal{K}'-J)^2\left(1+\frac{2}{\rho}\right) - J^2 + \frac{\tilde{a}^2+\tilde{b}^2}{\rho} - \frac{\beta(\tilde{a}^2+\tilde{b}^2)^2}{32(2-\beta)}$$

$$- \beta c^2(1-\beta/2) + \frac{\beta}{4}c(\tilde{b}^2-\tilde{a}^2) + \frac{\beta^2}{2\rho^2}\left|\frac{(\tilde{a}-i\tilde{b})^2}{2-\beta}+c\right|^2\} \ .$$

We may now pose the question as to how to choose the equilibrium parameters so as to optimize the stability properties. Clearly, and consistently with other experience we set

$$J = 0 \ .$$

Further it is clear that $\tilde{a} = 0$, $\tilde{b} \neq 0$ is optimum, giving the phase between the L = 1, L = 2 coils and that

$$c = \frac{\tilde{b}^2}{4(2-\beta)} \qquad (G)$$

for which

$$2\delta W = 2\pi\varepsilon^2\zeta^2 L[(\mathcal{K}')^2(2-\beta+2/\rho^2) + \tilde{b}^2/\rho^4 + 9\beta^2\tilde{b}^4/(128\rho^2(1-\beta/2)^2)$$

Thus by appropriate choice of L = 1, L = 2 fields one can obtain a system stabilized by wall effects to high order in the distortion parameter $\tilde{a}$. Of course one should, in principle also consider the important ε^4 terms omitted in the ordering. We note in passing that the above expression for δW also applies for the pure L = 2 system in which case we see that the system is unstable, as in the old Scyllac ordering but in opposition to the stellarator

ordering. Finally we note that a simple computation shows that (G), the condition for optimum stability, implies no $L = 2$ source fields outside the plasma.

Acknowledgment

The work described in this paper was supported by the U.S. Atomic Energy Commission under Contract No. AT(11-1)-3077.

Footnotes

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2. H. Weitzner, Paper CN28/J6, Fourth IAEA Conference on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, June 1971.
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